A PROPOSAL MATHEMATICAL MODEL OF GENERALIZED PARETO DISTRIBUTION FOR IP PACKET DELAY

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Abstract

Broadband multi-service traffic is transported over Internet Protocol (IP) networks, which are composed of different network nodes, physical links or sections. In practice cases, delay distribution of IP packet in each network component can be determined and expressed in a well-known mathematical function. It is proved in the paper that delay distribution of IP packet transported over the whole networks can be composed from delay components of generalized Pareto distribution and degenerate distribution by explicit mathematical model with certain hypotheses. The proposed model plays an important role in analyzing and evaluating performance, network planning or designing and traffic engineering for improving IP network performance.

Lưu lượng băng rồng đã dịch vụ được truyền tải qua môi trường mạng IP hop thành từ các nút mạng, các kết nối vật lý hoặc các phần đoạn mạng khác nhau. Trong thực tế, trả gói IP trong từng phần mạng có thể xác định và biểu diễn ở dạng hàm toán học phổ biến. Bài báo cũng minh chứng rằng phân bố trả gói IP qua toàn mạng có thể được tổng hợp từ các thành phần trả có phân bố Pareto tổng quát và trả tiến định thông qua các mô hình toàn học với các giả thiết nhất định. Mô hình phân bố trả dữ liệu xuất trong bài báo đồng với trừ quân trong việc phân tích và đánh giá hiệu năng, lập kế hoạch hoặc thiết kế mạng và là cơ sở cho các giải pháp kỹ thuật lưu lượng nhằm cải thiện hiệu năng mạng IP.

Index terms

IP packet, delay, probability distribution, multi-section network, convolution, Laplace transforms.

1. Introduction

Internet Protocol (IP) packet delay is one of key performance metrics of next generation transport networks based on IP/MPLS (Multi-Protocol Label Switching) technology. It is especially important for real-time interactive services such as IP television, voice and video over IP, internet game, etc. IP packet delay can be resulted from direct measurements \([1, 2]\) or by estimating moments of the total delay distribution from component ones \([3]\). However, the first method is difficult to be performed in case of multi-domain networks due to management policies from different network providers. The second one has limited accuracy due to approximations and expected distribution is not fully quantified. Moreover, IP packets can be

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routed and forwarded through different network components where IP delay distributions are
normally predetermined. Therefore, a method for composing overall delay distribution from
component ones is necessary for flexible calculations of network performance and minimizing
network operating and maintenance costs.

Previous research results [1, 4] show that in certain cases depending on node processing
features, link capacity and traffic conditions, one-way transit delay of IP packets through
a network node such as a router is well modeled in form of Pareto or generalized Pareto
distribution. Moreover, IP packet propagation delay on each physical link (link for short in
the paper) is deterministic with a fixed value depending on link distance and properties of the
communication channel. However, the IP packet delay distributions have not been specified
adequately in mathematical models yet in these researches.

As an approach in order to solve the limitations mentioned above, it is proved in the paper
that one-way delay distribution of IP packets transported over the whole networks (hereinafter
referred to as composed distribution) can be composed from component ones by explicit
mathematical models based on the generalized Pareto with certain assumptions.

2. A Composition of IP Packet Delay Distributions of Generalized Pareto

2.1. Methodology

An IP network path can be set up through \( N \) network components (including hosts, routers,
links, etc) or \( N \) network sections, each of which is a set of network components.

It is assumed that IP packet delays over each of \( N \) components or sections are mutually
independent and their probability distributions can be represented by explicit probability
density functions as called \( PDF_i(t) \). According to probability theory [5], due to additive
property of IP packet delay, probability density function of IP packet delay over the whole
network as called \( PDF_\Sigma(t) \) can be calculated by convolution (*) as follows:

\[
PDF_\Sigma(t) = (PDF_1 * PDF_2 * \cdots * PDF_N)(t)
\]  

\[
\Rightarrow \mathcal{L}\{PDF_\Sigma(t)\} = \prod_{i=1}^{N} \mathcal{L}\{PDF_i(t)\} \text{ or } PDF_\Sigma(t) = \mathcal{L}^{-1}\left\{\prod_{i=1}^{N} \mathcal{L}\{|PDF_i(t)|\}\right\}
\]  

where \( \mathcal{L}^{-1} \) is inverse Laplace transform. The cumulative distribution function of IP packet
delay over the whole network as called \( CDF_\Sigma(t) \) can be calculated from (2) as follows:

\[
CDF_\Sigma(t) = \int_{-\infty}^{t} PDF_\Sigma(x) \, dx = \int_{-\infty}^{t} \mathcal{L}^{-1}\left\{\prod_{i=1}^{N} \mathcal{L}\{|PDF_i(x)|\}\right\} \, dx
\]
2.2. A General Model

A general mathematical model is proposed for composing delay distribution of IP packet transported over the network path through either of the following:

- N nodes (routers) and N−1 inter-node links
- N sections and N−1 inter-section links

It can be assumed that IP packet delay on each link is a deterministic value of \( d_i \) for the \( i^{th} \) link [1]. This means that its probability density function as called \( PDF_{L_i}(t) \) and cumulative distribution function as called \( CDF_{L_i}(t) \) are represented by a variant of the degenerate distribution as follows:

\[
PDF_{L_i}(t) = \delta(t - d_i), \quad CDF_{L_i}(t) = u(t - d_i)
\]

(4)

where \( i \in [1, N-1], \ u(t) \) is the unit step function and \( \delta(t) \) is the Dirac delta function. Meanwhile, IP packet delay in each node or section is supposed to conform to generalized Pareto distribution [1, 7] with the probability density function of \( PDF_{N_i}(t) \) and the cumulative distribution function of \( CDF_{N_i}(t) \) for the \( i^{th} \) node or section as follows:

\[
PDF_{N_i}(t) = \frac{1}{c_i} \left( 1 + k_i \frac{t_i - l_i}{c_i} \right) \left( \frac{k_i}{c_i} \right)^{-1} e^{-\left( \frac{t}{k_i} \right)^{\frac{1}{k_i}}} ;
\]

\[
CDF_{N_i}(t) = \begin{cases} 
1 - \left( 1 + k_i \frac{t_i - l_i}{c_i} \right)^{-\frac{k_i}{k_i}} & \text{if } k_i \neq 0 \\
1 - e^{-\frac{t_i}{c_i}} & \text{if } k_i = 0
\end{cases}
\]

(5)

where \( i \in [1, N], c_i \in \mathbb{R}^+ \) is scale parameter, \( k_i \in \mathbb{R} \) is shape parameter and \( l_i \in \mathbb{R}^+ \) is location parameter. The supports of component distributions are defined as follows:

\[
S_i = \begin{cases} 
[l_i, l_i - \frac{c_i}{k_i}] & \text{if } k_i < 0 \\
[l_i, +) & \text{if } k_i = 0
\end{cases}
\]

(6)

Within the support of (6), it can be seen that \( u \left[ 1 + \frac{k_i}{c_i} (t - l_i) \right] = 1 \) and \( u(t - l_i) = 1 \). Therefore, with the assumption of \( k_i < 0 \) (due to limited support of the Laplace transform [6]), it can be transformed from (5) as follows:

\[
\mathcal{L} \{ PDF_{N_i}(t) \} = \mathcal{L} \left\{ \frac{1}{c_i} \left[ 1 + \frac{k_i}{c_i} (t - l_i) \right] \left( \frac{k_i}{c_i} \right)^{\frac{1}{k_i}} e^{-\left( \frac{t}{k_i} \right)^{\frac{1}{k_i}}} \times u \left[ 1 + \frac{k_i}{c_i} (t - l_i) \right] \times u(t - l_i) \right\} =
\]

\[
\frac{1}{k_i} \Gamma \left( \frac{1}{k_i} \right) \times e^{\left( \frac{c_i}{k_i} \times s \right)} \times e^{-\frac{c_i}{k_i} t_i}
\]

(7)

where \( \Gamma(x) \) is the gamma function. Moreover, Laplace transform of (4) is expressed as follows:

\[
\mathcal{L} \{ PDF_{L_i}(t) \} = e^{-d_i \times s}
\]

(8)
From (2), (7) and (8), it can be transformed as follows:

\[
PDF(t) = \mathcal{L}^{-1} \left\{ \prod_{i=1}^{N} \left[ \mathcal{L} \left( PDF_{N_i}(t) \right) \right] \times \prod_{i=1}^{N-1} \left[ \mathcal{L} \left( PDF_{L_i}(t) \right) \right] \right\} =
\]

\[
= \mathcal{L}^{-1} \left\{ (-1)^N \prod_{i=1}^{N} \left[ \frac{\Gamma \left( -\frac{1}{k_i} \right) \times (c_i)^{\left( \frac{1}{k_i} \right)} \times e^{\sum_{i=1}^{N} \left( \frac{c_i}{k_i} \right) \times x} \times e^{-\sum_{i=1}^{N-1} \left( l_{i} \times x \right) \times e^{-\sum_{i=1}^{N-1} \left( d_{i} \times x \right)}}}{\prod_{i=1}^{N} \left( \frac{c_i}{k_i} \right)^{-\sum_{i=1}^{N} \left( \frac{1}{k_i} \right)}} \right\} \times \mathcal{L}^{-1} \left\{ \frac{e^{\sum_{i=1}^{N} \left( \frac{c_i}{k_i} \right) \times x} \times e^{-\sum_{i=1}^{N-1} \left( l_{i} \times x \right) \times e^{-\sum_{i=1}^{N-1} \left( d_{i} \times x \right)}}}{s^{-\sum_{i=1}^{N} \left( \frac{1}{k_i} \right)}} \right\}
\]

(9)

Applying Laplace transform and its properties [8], (9) can be rewritten as follows:

\[
PDF(t) = (-1)^N \prod_{i=1}^{N} \left[ \frac{\Gamma \left( -\frac{1}{k_i} \right) \times (c_i)^{\left( \frac{1}{k_i} \right)} \times e^{\sum_{i=1}^{N} \left( \frac{c_i}{k_i} \right) \times x}}{\left( \frac{1}{k_i} \right)^{1+\frac{1}{k_i}}} \times \left( \sum_{i=1}^{N} \frac{c_i}{k_i} \right)^{-\sum_{i=1}^{N} \left( \frac{1}{k_i} \right)} \right] \times \mathcal{L}^{-1} \left\{ \frac{e^{\sum_{i=1}^{N} \left( \frac{c_i}{k_i} \right) \times x} \times e^{-\sum_{i=1}^{N-1} \left( l_{i} \times x \right) \times e^{-\sum_{i=1}^{N-1} \left( d_{i} \times x \right)}}}{\left( \sum_{i=1}^{N} \frac{c_i}{k_i} \right)^{-\sum_{i=1}^{N} \left( \frac{1}{k_i} \right)}} \right\}
\]

(10)

With parameters of (11) and a normalized coefficient of (12) introduced, (10) can be transformed into (13) as follows:

\[
c_{\Sigma} = \frac{\sum_{i=1}^{N} \frac{c_i}{k_i}}{\sum_{i=1}^{N} \left( \frac{1}{k_i} \right)}, \quad k_{\Sigma} = \frac{1}{\sum_{i=1}^{N} \left( \frac{1}{k_i} \right)}, \quad l_{\Sigma} = \sum_{i=1}^{N} l_i + \sum_{i=1}^{N-1} d_i
\]

(11)

\[
C = (-1)^{N+1} \prod_{i=1}^{N} \left[ \frac{\Gamma \left( -\frac{1}{k_i} \right) \times (c_i)^{\left( \frac{1}{k_i} \right)} \times e^{\sum_{i=1}^{N} \left( \frac{c_i}{k_i} \right) \times x}}{\left( \frac{1}{k_i} \right)^{1+\frac{1}{k_i}}} \times \left( \sum_{i=1}^{N} \frac{c_i}{k_i} \right)^{-\sum_{i=1}^{N} \left( \frac{1}{k_i} \right)}} \right] \times \frac{\left( \sum_{i=1}^{N} \frac{c_i}{k_i} \right)^{-\sum_{i=1}^{N} \left( \frac{1}{k_i} \right)}}{\sum_{i=1}^{N} \left( \frac{1}{k_i} \right)^{-\sum_{i=1}^{N} \left( \frac{1}{k_i} \right)}}
\]

(12)

\[
PDF_{\Sigma}(t) = \frac{1}{c_{\Sigma}} \times \left[ \frac{k_{\Sigma}}{c_{\Sigma}} \times (t - l_{\Sigma} + 1) \right]^{-(\frac{1}{k_{\Sigma}})-1} \times u \left[ \frac{k_{\Sigma}}{c_{\Sigma}} \times (t - l_{\Sigma} + 1) \right] \times u(t - l_{\Sigma})
\]

(13)

With the assumption of \( c_i > 0 \) and \( k_i < 0 \) \( \forall i \in [1, N] \) from (5) and (7), the support of the composed distribution can be defined based on (6) and sub-additive property of IP packet delay as follows:

\[
S_{\Sigma} = [l_{\Sigma}, (l_{\Sigma} - c_{\Sigma}/k_{\Sigma})]
\]

(14)
Within this support, (13) can be rewritten as follows:

\[ PDF_{\Sigma}(t) = \frac{1}{c_{\Sigma}} \times \left[ \frac{k_{\Sigma}}{c_{\Sigma}} \times (t - l_{\Sigma}) + 1 \right]^{-\left( \frac{1}{k_{\Sigma}} + 1 \right)} \]  

(15)

Applying mathematical basics [11], it can be calculated from (3) and (15) as follows:

\[ CDF_{\Sigma}(t) = \int_{l_{\Sigma}}^{t} PDF_{\Sigma}(x) \, dx = 1 - \left[ \frac{k_{\Sigma}}{c_{\Sigma}} \times (t - l_{\Sigma}) + 1 \right]^{-\frac{1}{k_{\Sigma}}} \]  

(16)

Mean \((\mu_{\Sigma})\) and variance \((\sigma_{\Sigma}^2)\) of the composed delay distribution can be computed based on their definitions [11] and (14), (15) as follows:

\[
\mu_{\Sigma} = \int_{l_{\Sigma}}^{(l_{\Sigma} - \frac{c_{\Sigma}}{k_{\Sigma}})} [t \times PPDF_{\Sigma}(t)] \, dt = l_{\Sigma} + \frac{c_{\Sigma}}{1 - k_{\Sigma}} = \sum_{i=1}^{N} l_i + \sum_{i=1}^{N-1} d_i + \frac{\sum_{i=1}^{N} c_i}{\sum_{i=1}^{N} \left( \frac{1}{k_i} \right) - 1},
\]

\[
\sigma_{\Sigma}^2 = \int_{l_{\Sigma}}^{(l_{\Sigma} - \frac{c_{\Sigma}}{k_{\Sigma}})} [(t - \mu_{\Sigma})^2 \times PPDF_{\Sigma}(t)] \, dt = \frac{c_{\Sigma}^2}{(1-k_{\Sigma})^2 \times (1-2k_{\Sigma})} = \frac{\sum_{i=1}^{N} \left( \frac{1}{k_i} \right) \times \left( \sum_{i=1}^{N} \frac{c_i}{k_i} \right)^2}{\left( \sum_{i=1}^{N} \frac{1}{k_i} \right)^2 \times \left( \sum_{i=1}^{N} \frac{1}{k_i} - 1 \right)^2}
\]  

(17)

2.3. A Special Model

A special case is considered for a network path through \(N\) nodes or sections in which IP packet delays are independent and identically distributed in form of generalized Pareto family with the following assumption:

\[ c_i = c, \ k_i = k, \ l_i = l, \ \forall i \in [1, N] \]  

(18)

in which: \(c \in \mathbb{R}^+, \ k < 0, \ l \in \mathbb{R}^+\) are scale, shape, location parameter respectively of the IP packet delay distribution in each node or section. Using (18), equations (11), (14), (15), (16), (17) can be rewritten as (19), (20), (21), (22), (23) respectively. The composed distribution parameters are resulted as:

\[ c_{\Sigma} = c, \ k_{\Sigma} = \frac{k}{N}, \ l_{\Sigma} = N \times l + d_{\Sigma}, \ d_{\Sigma} = \sum_{i=1}^{N-1} d_i \]  

(19)

The support of composed distribution is resulted as:

\[ S_{\Sigma} = \left\{ t \in \left( (N \times 1 + d_{\Sigma}), \ (N \times \left( 1 - \frac{c}{k} \right) + d_{\Sigma}) \right) \right\} \]  

(20)

The composed probability distribution functions are resulted as:

\[ PPDF_{\Sigma}(t) = \frac{1}{c} \times \left[ \frac{k}{N \times c} \times (t - N \times l - d_{\Sigma}) + 1 \right]^{-\left( \frac{N}{k} + 1 \right)} \]  

(21)
\[CDF_\Sigma(t) = 1 - \left(\frac{k}{N \times c} \times (t - N \times l - d) + 1\right)^\frac{N}{k} \]  

(22)

The moments of composed distribution are resulted as:

\[
\mu_\Sigma = N \times l + \frac{N \times c}{N - k} + d, \quad \sigma^2_\Sigma = \frac{N^3 \times c^2}{(N - 2k) \times (N - k)^2}
\]  

(23)

2.4. Typical Samples of the Model

Some specific sample data derived from the distribution model are computed, illustrated and compared in the Table 1 and Figures from 1 to 6. There are six case studies used to survey the composed distribution versus the variation of one or more component distribution parameters which are identical for every nodes in the case number from 1 to 4 and they are different for the remainder cases.

<table>
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<th>$N$</th>
<th>$l_i$</th>
<th>$c_i$</th>
<th>$-k_i$</th>
<th>$d$</th>
<th>$l_\Sigma$</th>
<th>$c_\Sigma$</th>
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Note: $d$: link delay; $l_i$, $c_i$, $\mu_\Sigma$ is in millisecond; $i \in [1, N]$ for $l_i$, $c_i$, $k_i$, $d$.

3. Evaluation and Discussion

Based on the model proposed in part 2, it can be seen that if the delay distribution of IP packets in each node belongs to generalized Pareto family and their delay on each link is deterministic then IP packet delay over a section composed of those nodes and links also
follow the generalized Pareto distribution as assumed in (5). The same rule can be recursively applied for the whole network composed of different sections or domains.

The composed delay distribution functions of (15), (16) with the support of (14) conform to power law with the distribution scale parameters of $\sigma_\Sigma$, the shape parameters of $k_\Sigma$ and the location parameters of $l_\Sigma$ as defined in (11). All of them are depended on a set of distribution parameters of delay components. Specifically, it can be affirmed of additive property of location parameters and inverse shape parameters. The location parameter of total delay distribution is equal to sum of delay on inter-node links and location parameters of component delay distributions in nodes. The inverse shape parameter of total delay distribution equals to sum of inverse shape parameters of component delay distributions in nodes. The scale parameter of total delay distribution is depended on both scale and shape parameters of component delay distributions in nodes for general cases. In the case of homogeneous delay distributions, it can be seen from (19) that shape parameter of total delay distribution is $N$.
times smaller than that of a component distribution while the scale parameters are the same for all of them.

It can be recognized from the mathematical model as well as from the data samples that the variations of these parameters have different effects on IP packet delay distribution. The value of $c_\Sigma$ determines statistical dispersion of delay distribution as follows: larger value causes distribution more spread out toward $+\infty$ and lower probability density, smaller value causes it more concentrated and higher density ($3^{rd}$ case). It is also inferred from (16) that:

$$CDF_\Sigma(t, c_\Sigma, k_\Sigma, l_\Sigma) = CDF_{\xi}(t/c_\Sigma, 1, k_\Sigma, l_\Sigma/c_\Sigma)$$  \hspace{1cm} (24)

The value of $l_\Sigma$ determines the location or shift of the distribution along the time axis. Larger location value causes distribution shifted more to the right and vice versa ($2^{nd}$ case). The value of $k_\Sigma$ affects the shape of a distribution rather than simply shifting it as the location parameter does or stretching/shrinking it as the scale parameter does ($4^{th}$ case).

It can be seen from (11), (15) and (16) and the second case study that deterministic delay components such as propagation delays on the links affect and contribute only to the location parameter of the composed distribution but not the others.
Finally, it is observed from (17) and table 1 that the variance of composed delay distribution is depended only on scale and shape parameter values of component ones in nodes but not on the value of location parameters and of IP packet delay on links while its mean value is depended on all of them.

4. Conclusion

A new mathematical model of generalized Pareto distribution has been established and proposed in the paper as a method for composing delay distributions of IP packet over the whole network from component ones. The problem has been resolved completely with the assumptions of negative shape parameter values and the independence of component delay distributions.

Typical model samples has also been analyzed, compared and evaluated to summarize qualitative and quantitative characteristics of IP packet delay distributions. It is concluded that composed distributions inherit properties of the component ones in the same family of generalized Pareto distribution. Different component distribution parameters have different effects and contributions on the composed one. The composed location parameter is the sum
of component ones and link delays. The inverse composed shape parameter is the inverse sum of component ones only. The composed scale parameter is generally depended on both component scale and shape parameters. The composed distribution mean is depended mostly on location parameters and link delays while its variance is depended on scale and shape parameters.

Further studies are needed for the case of mutually dependent distributions, positive value of the shape parameter, mixed distributions which cannot be well defined by explicit functions or other kinds of distributions matched in practice.

References


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