ON THE PERFORMANCE ANALYSIS OF UNDERLAY RELAY COGNITIVE NETWORKS WITH ERRONEOUS CHANNEL INFORMATION

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Abstract

This paper presents a general framework for performance analysis of underlay relay cognitive networks with arbitrary number of hops, erroneous Nakagami-$m$ fading channel information, secondary users (SUs) of different maximum transmit power (MTP) levels, maximum transmit power constraint (MTPC), and interference power constraint (IPC). Exact and asymptotic bit error rate (BER) formulas are proposed in closed-form and extensively corroborated by Monte-Carlo simulations. These formulas play a key role in system performance analysis under different operation parameters as well as system design optimization without the need of time-consuming and exhaustive simulations. Notably, analytic results show that underlay relay cognitive networks experience performance saturation and their performance is significantly affected by channel information imperfection, modulation level, and fading severity level.

Index terms
Relaying communications; underlay cognitive radio; Nakagami-$m$ fading; bit error rate.

1. Introduction

The conventional assignment of frequency bands by means of fixed licensees is not flexible and efficient, leading to witnessed spectrum scarcity for emerging modern wireless services [1]. The cognitive radio technology has effectively solved this problem of spectrum under-utilization by allowing SUs to opportunistically occupy empty frequency spectrum [2]. Thanks to this mechanism, SUs can still operate in the frequency band, which has been primarily allotted to primary users (PUs) without causing any severe interference to primary networks. Consequently, the cognitive radio technology can substantially improve the overall spectrum utilization.

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In general, cognitive radios are implemented to work in three different modes: interweave; overlay; and underlay [2]. This paper considers the underlay mode for its distinct feature on rather low implementation complexity [2]. In this mode, SUs must adaptively control its transmit power to maintain the corresponding induced interference below a certain threshold that the PUs can tolerate. Therefore, the transmission range of SUs is dramatically reduced. Nevertheless, with the aid of relaying communications techniques [3], which exploit shorter range communication for lower path loss, underlay cognitive networks can efficiently overcome the aforementioned limitation. As such, underlay relay cognitive networks, which combine both cognitive radio and relaying communications technologies, can achieve high spectrum utilization efficiency and wide radio range.

Channel state information (CSI) plays an important role in system design such as coherent detector optimization. Nevertheless, channel estimation algorithms still work at a certain accuracy, and hence the requirement of perfect CSI is inevitably impossible. The effect of imperfect channel information on the outage performance of underlay two-hop cognitive networks is investigated under consideration of Rayleigh fading in [2], [4]–[7], and Nakagami-\(m\) fading in [8], which is more flexible and includes the Rayleigh fading as a special case of \(m = 1\). Theoretically, the outage analysis is very helpful since it can provide useful insights in the information-theoretic performance limit and motivate code design to reach it. However, effective determination of this limit is unfortunately rather problematic. Meanwhile, the BER analysis provides the real — not a limit — system performance for a targeted spectral efficiency (i.e., modulation level) and therefore, it is of greater practical importance. In spite of the undoubted usefulness of underlay multi-hop cognitive networks (UMHCNs), only limited analytic investigations on the BER analysis of these systems taking into the account erroneous channel information\(^1\) have been reported in the open technical literature [9]–[11]. However, these works only investigate Rayleigh fading channels and SUs of the same MTP level.

Motivated by the above, this paper aims to provide a general framework for the BER analysis of UMHCNs under quite practical conditions concurrently: arbitrary number of hops, imperfect Nakagami-\(m\) fading channel information on all the involved channels, SUs of different MTP levels, IPC, and MTPC. Both asymptotic and exact analyses are investigated. Notably, our analyses include previous works as special cases. For instances, the BER of conventional multi-hop systems with perfect CSI in [13] is a special case of our asymptotic results for large maximum interference power (MIP) and no channel estimation error while our asymptotic results for large MTP can be used to determine the BER of UMHCNs under only IPC, such as [9]. Likewise, various results illustrate that UMHCNs suffer the error floor phenomenon and their performance is substantially impacted by CSI imperfection, modulation level, and fading severity level.

\(^1\)It is noted that [12] analyses the BER of UMHCNs with perfect Nakagami-\(m\) fading channel information.
2. System Model

Fig. 1 shows the system model for the underlay $N$-hop cognitive network under investigation. Relaying communications is realised in the secondary network where $(N - 1)$ intermediate SUs $\{U_1, ..., U_{N-1}\}$ assist the transmission between $U_0$ and $U_N$, interfering with the PU, $P_{Rx}$. We consider non-identical independent frequency-flat Nakagami-$m$ fading channels. To this end, the channel gain between the transmitter $U_u$ and the receiver $U_v$, $|h_{u,v}|$, is Nakagami-$m$ distributed with the parameter pair $\{m_{u,v}, \lambda_{u,v}\}$, i.e., the probability density function (pdf) of $|h_{u,v}|$ is expressed as $f_{|h_{u,v}|}(x) = 2\Lambda^{m_{u,v}}_{u,v} x^{2m_{u,v}-1}e^{-\Lambda_{u,v} x^2}/\Gamma(m_{u,v})$ where $x > 0$, $\Gamma(\cdot)$ is the Gamma function [14, eq. (8.310.1)], and $\Lambda_{u,v} = m_{u,v}/\lambda_{u,v}$ with $\lambda_{u,v} = E\{|h_{u,v}|^2\} = d_{u,v}^{-\alpha_{u,v}}$. Also, $E\{\cdot\}$ denotes the expectation operator while $d_{u,v}$ and $\alpha_{u,v}$ denote the distance between two users and the involved path-loss exponent, respectively; $u \in \{0, 1, \ldots, N-1\}$ and $v \in \{1, 2, \ldots, N, L\}$, with $L$ denoting the index of $P_{Rx}$, [15].

It is also recalled that $N$-hop wireless transmission takes places in $N$ phases. In the $n^{th}$ phase, the transmitter $U_u$ sends a group of $q$ modulated symbols $x_u = [x_u(1), x_u(2), \ldots, x_u(q)]$, each with symbol energy, $P_u = E\{|x_u(t)|^2\}$, where $t$ denotes the time index. Based on this, the symbol group is demodulated by the receiver $U_n$ and re-modulated as $x_n = [x_n(1), x_n(2), \ldots, x_n(q)]$, each with the symbol energy, $P_n$. Then, they are forwarded to the SU $U_{n+1}$ during the $(n+1)^{th}$ phase$^2$. To this effect, the received signal in the $n^{th}$ phase can be modelled as follows:

$$y_{u,n} = h_{u,n} x_u + z_{u,n},$$

where $y_{u,n}$ is the signal received at $U_n$ from $U_u$ with $u = n - 1$ and $z_{u,n} \sim CN(0, \mathcal{N}_n)$ $^3$.

$^2$For the purpose of notation simplicity and to avoid any confusion, the time index is therefore ignored.

$^3z \sim CN(\kappa, \omega)$ denotes a circular symmetric complex Gaussian random variable with mean $\kappa$ and variance $\omega$. 
represents the additive noise at \( U_n \).

It is recalled here that operating in the underlay mode, as in e.g., [2], the SU \( U_u \) is required to set its transmit power according to \( P_u = \min(\mathcal{Q}/|h_{u,L}|^2, \mathcal{P}_{um}) \) for the maximum transmission range and for meeting both the corresponding IPC, \( P_u \leq \mathcal{Q}/|h_{u,L}|^2 \), and MTPC, \( P_u \leq \mathcal{P}_{um} \). Here, \( \mathcal{Q} \) is the MIP that can be tolerated by PU whereas \( \mathcal{P}_{um} \) is the MTP designed for the SU \( U_u \). Also, in order to set the transmit power of SUs, the channel coefficient \( h_{u,L} \) must be available at SUs. Since the channel estimation error is inevitably unavoidable, the investigation of its effect on the system performance is urgent and essential.

Based on the linear minimum mean-square error channel estimation, the corresponding estimation error model can be expressed as [16],

\[
\hat{h}_{u,v} = h_{u,v} + \xi_{u,v},
\]

where \( \xi_{u,v} \sim \mathcal{CN}(0, \tau_{u,v}) \) is the channel estimation error, \( h_{u,v} \) is the estimate of the \( u-v \) channel which follows the Nakagami-\( m \) distribution with parameters \( \{m_{u,v}, \zeta, \psi, u,v \} \), whereas \( \xi_{u,v} \) is statistically independent of \( \hat{h}_{u,v} \). Furthermore, the corresponding variances of \( \xi_{u,v} \) and \( h_{u,v} \) are expressed as \( \tau_{u,v} = \lambda_{u,v}/(1 + \rho_{u,v} \theta_{u,v} \lambda_{u,v}) \) and \( \zeta_{u,v} = \lambda_{u,v} - \tau_{u,v} = \rho_{u,v} \theta_{u,v} \lambda_{u,v}^2/(1 + \rho_{u,v} \theta_{u,v} \lambda_{u,v}) \), respectively [16]. Here, \( \rho_{u,v} > 0 \) accounts for the quality of the estimator and \( \theta_{u,v} \) denotes the transmit signal-to-noise ratio (SNR). Following [8], we select the setting of \( \theta_{u,v} = \mathcal{Q}/(\mathcal{N}_v \lambda_{u,v}) \) in this paper.

To this effect, by letting \( g_{u,v} = |\hat{h}_{u,v}|^2 \), it immediately follows that the corresponding pdf is given by [17, eq. (2.21)]

\[
f_{g_{u,v}}(x) = \beta_{u,v}^m u^m e^{−\beta_{u,v} u}/\Gamma(m_{u,v}),
\]

where \( x > 0 \) and \( \beta_{u,v} = m_{u,v}/\zeta_{u,v} \).

3. Exact Error Rate Analysis

By substituting (2) into (1) one obtains \( y_{u,n} = \hat{h}_{u,n} x_u + (\xi_{u,n} x_u + z_{u,n}) \). When the perfect CSI is not available (i.e., only \( \hat{h}_{u,L} \) is available), the SU \( U_u \) is required to modify its transmit power as \( \hat{P}_u = \min(\mathcal{Q}/|\hat{h}_{u,L}|^2, \mathcal{P}_{um}) \). Based on this transmit power setting, the instantaneous SNR in the \( n^{th} \) phase is straightforwardly expressed as

\[
\psi_{u,n} = \frac{\mathcal{E}\{ |\hat{h}_{u,n} x_u |^2 \}}{\mathcal{E}\{ |\xi_{u,n} x_u + z_{u,n} |^2 \}} = \frac{\min(\mathcal{Q}/g_{u,L}, \mathcal{P}_{um}) g_{u,n}}{\min(\mathcal{Q}/g_{u,L}, \mathcal{P}_{um}) \tau_{u,n} + \mathcal{N}_n}. \tag{4}
\]

**Theorem 1.** The cumulative distribution function (cdf) of \( \psi_{u,n} \) is given by

\[
F_{\psi_{u,n}}(y) = \frac{\Gamma(m_{u,L}; \beta_{u,L} \mu_u)}{\Gamma(m_{u,L})} + \frac{\gamma(m_{u,n}; \beta_{u,n} y (\tau_{u,n} + \hat{P}_{um})) \gamma(m_{u,L}; \beta_{u,L} \mu_u)}{\Gamma(m_{u,n}) \Gamma(m_{u,L})} \left( \beta_{u,n} y \right)^i \mathcal{Q}^i \tau_{u,n} \Gamma(p + m_{u,L}; \left( \beta_{u,L} + \beta_{u,n} \mathcal{Q} y \right) \mu_u) \right) \left( \beta_{u,L} + \beta_{u,n} \mathcal{Q} y \right)^{p+m_{u,L}}
\]

\[
- \frac{\beta_{u,L}^m u^m e^{−\beta_{u,L} u} m_{u,n}−1 i^i \Gamma(p+m_{u,L}; \left( \beta_{u,L} + \beta_{u,n} \mathcal{Q} y \right) \mu_u) \Gamma(m_{u,L})}{\Gamma(m_{u,L})} \sum_{i=0}^{m_{u,n}} \sum_{p=0}^{m_{u}-1} \frac{\beta_{u,n} y \Gamma(p+m_{u,L}; \left( \beta_{u,L} + \beta_{u,n} \mathcal{Q} y \right) \mu_u)}{i!} \left( \beta_{u,n} y \right)^i \mathcal{Q}^i \tau_{u,n} \Gamma(p + m_{u,L}; \left( \beta_{u,L} + \beta_{u,n} \mathcal{Q} y \right) \mu_u)
\]

\[
\tag{5}
\]
where $\mu_u = Q/P_{um}$, $\bar{P}_{um} = N_n/P_{um}$, $\bar{Q} = N_n/Q$, $\gamma (a; x) \triangleq \int_0^x t^{a-1} e^{-t} dt$ and $\Gamma (a; x) \triangleq \int_x^{\infty} t^{a-1} e^{-t} dt$ denote the “lower” and “upper” incomplete gamma functions in [14, eq. (8.350.1)] and [14, eq. (8.350.2)], respectively.

**Proof:** Please see Appendix A.

The BER analysis can be completed in two stages:

- **Stage 1:** Computation of the accurate closed-form average BER, $P_e(n)$, in the $n^{th}$ phase.
- **Stage 2:** Use of $P_e(n)$ for $n = 1, 2, ..., N$ in Stage 1 to compute the average BER of the underlay $N$-hop cognitive networks with the help of [13, eq. (9)], namely,

$$P_e = \sum_{n=1}^{N} \left[ P_e(n) \prod_{j=n+1}^{N} (1 - 2P_e(j)) \right].$$

(6)

It is undoubted that Stage 1 is the most important stage. The average BER in the $n^{th}$ phase is computed with the aid of the pdf of $\psi_{u,n}$, $f_{\psi_{u,n}}(x)$, and the corresponding instantaneous BER. In more details, given the instantaneous BERs for square $M$-ary Quadrature Amplitude Modulation ($M$-QAM) with $M = 2^l$ ($l$ even) as $2\Psi (\sqrt{M}, g, M; x)$ and rectangular $M$-QAM with $M = 2^l$ ($l$ odd) as $\Psi (I, r, M; x) + \Psi (J, r, M; x)$ in [18, eq. (16)] and [18, eq. (22)], correspondingly, the $P_e(n)$ can be expressed as follows:

$$P_e(n) = \left\{ \begin{array}{ll}
\int_0^{\infty} \{ \Psi (I, r, M; x) + \Psi (J, r, M; x) \} f_{\psi_{u,n}}(x) \, dx , & l \text{ odd} \\
2 \int_0^{\infty} \Psi (\sqrt{M}, g, M; x) f_{\psi_{u,n}}(x) \, dx , & l \text{ even}
\end{array} \right. $$

(7)

where

$$\Psi (s, q, M; x) \triangleq \frac{2}{\log_2 M} \sum_{i=0}^{\log_2 s} (1-2^{-k})_{s-1} \frac{(-1)^{i2^{k-1}/s} Q\left(\sqrt{(2i+1)^2 qx}\right)}{(2^{k-1} - [i2^{k-1}/s + 0.5])^{-1}}$$

and $g = 3/(M - 1)$, $r = 6/(I^2 + J^2 - 2)$, $I = 2^{(l-1)/2}$, $J = 2^{(l+1)/2}$. Also, $\lfloor.\rfloor$ and $Q(.)$ are the floor function and the $Q$-function [17], respectively. Obviously, the derivation of an exact closed-form expression for $P_e(n)$ is subject to analytical evaluation of the two infinite integrals in (7). Towards this end, by substituting (8) into (7) one obtains the following closed-form expression for the BER in the $n^{th}$ phase,

$$P_e(n) = \left\{ \begin{array}{ll}
\Phi (I, r, M; \chi) + \Phi (J, r, M; \chi) , & l \text{ odd} \\
2\Phi (\sqrt{M}, g, M; \chi) , & l \text{ even}
\end{array} \right. $$

(9)

The derivation of the average BER for other modulation schemes such as $M$-PSK (Phase-Shift Keying) can be proceeded in the same manner.
where
\[
\Phi (s, q, M; \chi) = \frac{2}{s \log_2 M} \sum_{k=1}^{\log_2 s} \sum_{i=0}^{s-1} (-1)^{\left\lfloor i 2^{k-1} / s \right\rfloor} \zeta \left((2i + 1)^2 q; \chi\right) / (2^{k-1} - i 2^{k-1} / s + 0.5) + i,
\]
and \( \chi = \{m_{u,L}, \beta_{u,L}, \mu_u, m_{u,n}, \beta_{u,n}, \tau_{u,n}, Q, P_{um}\} \) is a set of parameters.

Also, the \( \zeta (\beta; \chi) \) function in (10) is defined as:
\[
\zeta (\beta; \chi) = \int_0^\infty Q \left( \sqrt{\beta x} \right) f_{\psi_u,n} (x) dx.
\]

**Theorem 2.** The exact closed form of \( \zeta (\beta; \chi) \) is represented as
\[
\zeta (\beta; \chi) = \frac{\Gamma (m_{u,L}; \beta_{u,L} \mu_u)}{\Gamma (m_{u,L})} I_1 + \frac{\gamma (m_{u,L}; \beta_{u,L} \mu_u)}{\Gamma (m_{u,n}) \Gamma (m_{u,L}) \sqrt{2\pi}} I_3
\]
\[\begin{align*}
&- \frac{\beta^{m_{u,L}}}{\Gamma (m_{u,L}) \sqrt{2\pi}} \sum_{i=0}^{m_{u,n}-1} \sum_{p=0}^i \frac{1}{i!} \left( \frac{1}{p} \right) \left( \frac{\beta_{u,n} \tau_{u,n}}{\beta} \right)^i \left( \frac{Q}{\tau_{u,n}} \right)^p I_2
\end{align*}\]
where
\[
I_1 = 0.5
\]
\[
I_2 = \frac{\Gamma (p + m_{u,L})}{e^{\beta_{u,L} \mu_u}} \sum_{c=0}^{p + m_{u,L} - 1} \frac{\mu_u^c}{c!} \gamma (i, \tau_{u,n} + P_{um}, p + m_{u,L} - c)
\]
\[
I_3 = \Omega (m_{u,n}, \beta_{u,n} (\tau_{u,n} + P_{um}) / \beta, 0.5)
\]

\[
\gamma (i, a, b, k, j) = \frac{b^{i+j+\frac{1}{2}k-\frac{1}{2}}}{2\Gamma (j)} \Gamma \left(i + \frac{1}{2}\right) \Gamma \left(-i + j - \frac{1}{2}\right) 1_{F_1} \left(i + \frac{1}{2}; -i + j + \frac{3}{2}; \frac{ab}{k}\right)
\]
\[+ \frac{a^{i+j-\frac{1}{2}k-j}}{2} \Gamma \left(i - j + \frac{1}{2}\right) 1_{F_1} \left(j; -i + j + \frac{1}{2}; \frac{ab}{k}\right),
\]
\[
\Omega (m, a, b) = \Gamma (m) \left(\sqrt{\pi / 4b} - \sum_{i=0}^{m-1} a^i (2i-1)!! \sqrt{\pi / a + b}\right)
\]

with \( 1_{F_1} (x; y; z) \) being the Kummer confluent hypergeometric function defined in [14, eq. (9.210.1)].

**Proof:** Please see Appendix B.

Obviously, by making the necessary change of variables in (12) and substituting in (10), a closed-form formula for the BER in the \( n^{th} \) phase is deduced. To the best of our knowledge, (12) is novel.
4. Asymptotic Error Rate Analysis

The asymptotic error rate analysis is carried out by considering two extreme scenarios: i) the large maximum transmit power; ii) the large maximum interference power.

**Theorem 3.** For large maximum transmit power, (12) reduces to

\[
\lim_{\mathcal{P}_{um} \to \infty} \zeta (\beta; \chi) = \frac{1}{2} - \frac{\beta_{u,L}^{m_{u,L}}}{\Gamma (m_{u,L}) \sqrt{2\pi}} \sum_{i=0}^{m_{u,n}-1} \sum_{p=0}^{i} \frac{\Gamma (p + m_{u,L})}{i!} \left( \frac{\beta_{u,n} \tau_{u,n}}{\beta} \right)^i \left( \frac{\bar{Q}}{\tau_{u,n}} \right)^p \\
\times \gamma (i + 0.5; \beta_{u,L}, \beta_{u,n} \bar{Q}/\beta + m_{u,L}).
\]

(18)

**Proof:** If \( \mathcal{P}_{um} \to \infty \), then \( \mu_u \to 0 \) and \( \bar{P}_{um} \to 0 \). Therefore, the first term in (12) becomes 0.5, and \( \mathcal{I}_2 \) in the third term of (12) reduces to

\[
\Gamma (p + m_{u,L}) \gamma (i, \beta_{u,n} \tau_{u,n}/\beta + 0.5, \beta_{u,L}, \beta_{u,n} \bar{Q}/\beta + m_{u,L}).
\]

Since \( \Gamma (m_{u,L}; \beta_{u,L} \mu_u) \to \Gamma (m_{u,L}; 0) = \Gamma (m_{u,L}) \) and \( \Gamma (p + m_{u,L}; \beta_{u,L} + \beta_{u,n} \bar{Q}/\beta) \mu_u) \to \Gamma (p + m_{u,L}; 0) = \Gamma (p + m_{u,L}) \). Also, the second term in (12) becomes 0 since \( \gamma (m_{u,L}; \beta_{u,L} \mu_u) \to \gamma (m_{u,L}; 0) = 0 \). As a result, (18) consists of the first and third terms of (12), completing the proof.

Evidently, the value of (18) is unchanged according to \( \bar{Q} \). This clearly indicates that the network experiences the performance saturation for large values of \( \mathcal{P}_{um} \).

**Theorem 4.** For large maximum interference power, (12) becomes

\[
\lim_{\bar{Q} \to \infty} \zeta (\beta; \chi) = \Omega (m_{u,n}, \beta_{u,n} \bar{P}_{um}/\beta, 0.5) / \Gamma (m_{u,n}) / \sqrt{2\pi}.
\]

(19)

**Proof:** If \( \bar{Q} \to \infty \), then \( \mu_u \to \infty \), \( \bar{Q} \to 0 \), and \( \tau_{u,n} \to 0 \). Therefore, the first and third terms in (12) become 0 since \( \Gamma (m_{u,L}; \beta_{u,L} \mu_u) \to \Gamma (m_{u,L}; \infty) = 0 \) and \( \bar{Q} \to 0 \), respectively. As such, only the second term with \( \tau_{u,n} \to 0 \) is left in \( \zeta (\beta; \chi) \). Due to \( \gamma (m_{u,L}; \beta_{u,L} \mu_u) \to \gamma (m_{u,L}; \infty) = \Gamma (m_{u,L}) \), the second term becomes (19), completing the proof.

Notably, the result in (19) shows that when conditioned on \( \mathcal{P}_{um} \), the value of (19) remains constant and hence, the network suffers from the error floor for large values of \( \bar{Q} \).

Briefly, the asymptotic BER analysis is important and exposes the following useful insights on the performance of underlay relay cognitive networks:

- Underlay relay cognitive networks significantly suffer from the performance saturation at either the large maximum transmit power or the large maximum interference power.
• The asymptotic analysis can be straightforwardly extended to include some previous works as special cases:
  
  – **Special case 1: Only interference power constraint**
    In the present work, both MTPC and IPC are investigated. Some other works such as [9] only investigate the IPC. Obviously, our asymptotic analysis is applicable to determine the BER formula for UMHCNs under only the IPC. Indeed, the case of only IPC corresponds to our asymptotic analysis as $P_{um} \to \infty$, and hence, the BER for this case is computed by using (18), (10), (9), and (6), sequentially. Notably, even though the MTPC is relaxed, the BER formula for these networks has not been reported yet.

  – **Special case 2: Traditional multi-hop systems with perfect CSI**
    Traditional multi-hop systems with perfect CSI such as [13] is actually the UMHCN without the IPC and the channel estimation error. As such, by applying our asymptotic analysis for $Q \to \infty$ and $\tau_{u,n} = 0$, the BER formula for these systems is straightforwardly obtained. Importantly, this BER can be computed with the help of (19), (10), (9), and (6), sequentially.

5. **Illustrative Results**

This section illustrates the application of the derived formulas in evaluating the performance of UMHCNs in different key parameters without the need of time-consuming simulations. For illustration purpose, we take an example of the 3-hop communication scenario with the coordinates of each user being arbitrarily selected as follows: $U_0$ at $(0,0)$, $U_1$ at $(0.6,0.2)$, $U_2$ at $(0.8,0.3)$, $U_3$ at $(1,0)$, and the PU $P_{Rx}$ at $(0.7,0.5)$. To limit case-studies, we assume that noise variances at secondary receivers are normalized to be identical i.e., $N_{n} = N_0$ for any $n$; all channel estimators have same $\rho_{u,v} = \rho$ for any $\{u,v\}$; all channels have same path-loss exponent, $\alpha_{u,v} = 3$, and same fading severity parameter, $m_{u,v} = m = \{1,2,3\}$, for any $\{u,v\}$; the same MTP level for all secondary transmitters, i.e., $P_{um} = P_m$ with $u = \{0,1,2\}$. Also, we investigate two typical modulation formats, namely, 2-QAM for $l$ odd and 4-QAM for $l$ even. In all following figures, ‘Ana.’, ‘Sim.’, ‘Perf.’, and ‘Erro.’ stand for ‘Analysis’, ‘Simulation’, ‘Perfect CSI’ and ‘Erroneous CSI’, respectively.

Fig. 2 demonstrates the performance behaviour of the UMHCN with respect to the variation of $P_{m}/N_0$ for a fixed $Q/N_0$ at 15 dB, 2-QAM modulation, and $\rho = 0.9$. It is seen that the exact results in (12) are in excellent agreement with the corresponding results from Monte-Carlo simulations. Likewise, the offered asymptotic results in (18) match perfectly with the exact results for $P_{m}/N_0 \geq 15$ dB. It is also observed that UMHCNs substantially suffer from the performance saturation phenomenon and this performance saturation level is the BER of UMHCNs imposed by only the IPC - as discussed in Section 4. It should be emphasized here that the performance saturation comes from the fact that the power of the secondary
Figure 2. BER versus $P_m/N_0$ ($Q/N_0 = 15$ dB).

Figure 3. BER versus $P_m/N_0$ ($Q/N_0 = 15$ dB).
transmitters is constrained by the minimum value of the maximum transmit power, \( P_m \), and the maximum interference power, \( Q \). As a consequence, for the values of \( P_m \) over a certain value (e.g., about 15 dB in Fig. 2), the corresponding transmit power is totally determined by \( Q \), resulting in unchanged BER levels for any increase of \( P_m \). Moreover, channel information imperfection significantly deteriorates the BER performance. It is also shown that the system performance is dramatically improved, as expected, with respect to better fading conditions (i.e., the increase of \( m \)).

Under almost same operation conditions as Fig. 2 except the 4-QAM modulation, Fig. 3 demonstrates that the analysis well agrees with the simulation and the 4-QAM modulation experiences the same performance trend as the 2-QAM modulation. Also, the increase of modulation level, as expected, drastically degrades the system performance.

Fig. 4 shows the BER performance with respect to the change of \( Q/N_0 \) while \( P_m/N_0 \) is fixed at 15 dB for 2-QAM modulation and \( \rho = 0.9 \). It is clearly seen that the analytical results coincide with the corresponding simulation results, which verifies the accuracy of (12). Moreover, this figure exposes the performance improvement with \( Q \) for low-to-moderate values of \( Q \) (e.g., \( Q/N_0 < 40 \) dB). This is rather reasonable since \( Q \) upper-bounds the power of secondary transmitters and hence, the higher \( Q \) results in the higher transmit power which ultimately reduces the corresponding BER levels. For large \( Q \) (e.g., \( Q/N_0 \geq 40 \) dB), the error floor is observed and this error floor level is the BER of the traditional 3-hop system as reasoned in Section 4. The interpretation of this behaviour is the same as in the depicted

\[
\text{Figure 4. BER versus } Q/N_0 \ (P_m/N_0 = 15 \text{ dB}).
\]
scenario in Fig. 2. Furthermore, it is shown that the performance degradation due to the channel estimation error is substantially severe at small values of $Q/N_0$ and this performance degradation vanishes rapidly as $Q/N_0$ increases. This phenomenon emerges from the fact that the variance of the channel estimation error, $\tau_{u,n}$, is inversely proportional to $Q/N_0$, and hence, the larger the $Q/N_0$, the smaller the $\tau_{u,n}$ and the faster the system performance with imperfect CSI approaches that with perfect CSI. As a result, the asymptotic performance for both scenarios of imperfect and perfect CSI approaches the exact performance at large values of $Q/N_0$ and does not depend on the channel estimation error level. Likewise, the system performance is significantly better with respect to the increase in $m$, as expected.

In the same context but with the 4-QAM modulation, Fig. 5 shows that the analysis and the simulation are in perfect agreement and the performance behaviours for both 2-QAM and 4-QAM modulations are identical but the former is superior to the later, as expected.

The above results validate the accuracy of the derived formulas. Therefore, in the sequel, only analytic results are illustrated for time-savings. Figs. 6–7 expose the performance behaviour versus the channel estimation quality for $P_m/N_0 = 20$ dB, $Q/N_0 = 15$ dB, {2-QAM and 4-QAM} modulation. It is recalled that the variance of the channel estimation error $\tau_{u,v}$ is inversely proportional to $\rho_{u,v}$ and hence, by changing $\rho_{u,v}$ we can investigate the impact of the channel information imperfection on the system performance. As seen in Figs. 6–7, the BER performance of UMHCNs is substantially improved with the better channel estimation quality (i.e., the increase of $\rho_{u,v}$) for small values of $\rho < 5$ and quickly approaches that with
Figure 6. BER versus $\rho$ ($P_m/N_0 = 20\, dB, Q/N_0 = 15\, dB$).

Figure 7. BER versus $\rho$ ($P_m/N_0 = 20\, dB, Q/N_0 = 15\, dB$).
perfect CSI for large values of $\rho > 5$. Moreover, the fading severity, i.e. $m$, and the modulation level drastically degrade the system performance.

### 6. Conclusion

Exact and asymptotic BER analysis of underlay DF multi-hop cognitive networks under quite general conditions such as interference power constraint, maximum transmit power constraint, imperfect Nakagami-$m$ fading channel information, different maximum transmit power levels, and arbitrary number of hops was proposed in this paper and validated with extensive Monte-Carlo simulations. Notably, the derived asymptotic formulas can be efficiently used to compute the BER of underlay DF multi-hop cognitive networks under only interference power constraint and the BER of traditional DF multi-hop networks. Various results illustrate that underlay DF multi-hop cognitive networks ultimately incur the performance saturation phenomenon and their performance is substantially degraded by the channel estimation error, the modulation level, and the fading severity level.

### Appendix A

This appendix proves (5). Given $\psi_{u,n}$ in (4), it follows that

$$F_{\psi_{u,n}}(y) = \Pr\{\psi_{u,n} < y\} = \int_0^\infty \Pr\left\{g_{u,n} < \frac{\min \left(\frac{Q}{x}, \mathcal{P}_{um}\right) \tau_{u,n} + N_n}{y^{-1} \min \left(\frac{Q}{x}, \mathcal{P}_{um}\right)} \right\} f_{g_{u,L}}(x) \, dx. \quad (20)$$

Since the distribution of $g_{u,n}$ follows (3), $\Pr\{g_{u,n} < x\} = \frac{\gamma(m_{u,n}; \beta_{u,n}x)}{\Gamma(m_{u,n})}$. Using this result and (3), it immediately follows that

$$F_{\psi_{u,n}}(y) = \int_0^\infty \frac{\gamma(m_{u,n}; \beta_{u,n}y(\tau_{u,n} + Qx))}{\Gamma(m_{u,n})} f_{g_{u,L}}^{-1}(x) \, dx + \int_0^{\mu_u} \frac{\gamma(m_{u,n}; \beta_{u,n}y(\tau_{u,n} + \mathcal{P}_{um}))}{\Gamma(m_{u,n})} f_{g_{u,L}}^{-1}(x) \, dx.$$  

By performing the necessary change of variables in [14, eq. (8.352.1)] and substituting in the above, one obtains (5), completing the proof.

### Appendix B

This appendix proves (12). By integrating once by parts, $\zeta(\beta; \chi)$ in (11) can be re-written as

$$\zeta(\beta; \chi) = -\frac{1}{\sqrt{2\pi}} \int_0^\infty F_{\psi_{u,n}} \left(\frac{t^2}{\beta}\right) e^{-\frac{t^2}{2}} \, dt.$$  

Substituting (5) in $\zeta(\beta; \chi)$, one obtains (12)
where \( \mathcal{I}_1 = \frac{1}{\sqrt{2\pi}} \int_0^\infty e^{-\frac{t^2}{2}} dt \), \( \mathcal{I}_2 = \int_0^\infty t^2 \Gamma \left( p+m_{u,L} \right) \left( \frac{1}{\beta_{u,L}^2+P_u} \right) e^{-\frac{(\beta_{u,L}+\beta_{u,n} P_{Q}^2/\beta) t^2}{2}} dt \), \( \mathcal{I}_3 = \int_0^\infty e^{-\frac{(m_{u,n} + \bar{P}_{um}) t^2}{2}} dt \). Obviously, the proof is completed if \( \mathcal{I}_1, \mathcal{I}_2, \) and \( \mathcal{I}_3 \) are numerically evaluated as (13), (14), and (15), respectively. Towards this end, it is firstly noted that \( Q(x) \triangleq \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{t^2}{2}} dt \), and thus \( \mathcal{I}_1 = Q(0) = 0.5 \). Then, by recalling the alternative closed-form representation for \( \Gamma \left( a; x \right) \) in [14, eq. (8.352.2)] and performing the necessary change of variables, one obtains \( \mathcal{I}_2 \) as (14) where the \( \Upsilon \left( i, a, b, k, j \right) \) function is defined as \( \Upsilon \left( i, a, b, k, j \right) = \int_0^\infty \frac{e^{ax} - e^{bx}}{\beta(kx^2)^j} dx \). After some basic manipulations, \( \Upsilon \left( i, a, b, k, j \right) \) has the closed-form as (16). Finally, the \( \mathcal{I}_3 \) integral is expressed in closed-form as (15) where the \( \Omega \left( m, a, b \right) \) function is given by \( \Omega \left( m, a, b \right) = \int_0^\infty \gamma \left( m; at^2 \right) e^{-bt^2} dt \). By applying [14, eq. (8.352.1)], one rewrites it as \( \Omega \left( m, a, b \right) = \int_0^\infty \Gamma \left( m \right) \left( 1 - e^{-at^2} \sum_{i=0}^{m-1} \frac{a_i t^{2i}}{i!} \right) e^{-bt^2} dt = \Gamma \left( m \right) \left( \int_0^\infty e^{-bt^2} dt - \sum_{i=0}^{m-1} \frac{a_i t^{2i}}{i!} \int_0^\infty t^{2i} e^{-\left( a+b \right) t^2} dt \right) \). The \( \Omega \left( m, a, b \right) \) integral can be solved in closed-form as (17) by recalling that \( \int_0^\infty e^{-at^2} dt \triangleq \sqrt{\frac{\pi}{2a}} \) and applying [14, eq. (3.461.2)]. Therefore, the proof is completed.

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**References**


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