A GENERAL COORDINATE FORMULA
FOR DESIGNING PHASED ARRAY ANTENNAS
IN CYLINDRICAL SHAPE
WITH TRIANGULAR GRID

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Abstract

This paper proposes a mathematical model of coordinate combination in a particular shape of phased array antennas, namely, the cylindrical arrays with triangular grid (CATG), which are used in sonar applications. With the proposed solution, phase distribution in the CATG can be synthesized mathematically to calculate the radiation patterns when the main beam needs to be steered to any desirable direction. The results are calculated in both cases of with and without mutual coupling. The validity of the solution is evaluated by comparing the derived radiation patterns with that resulted by the tool Sensor Array Analyzer of MATLAB. However, the tool does not provide the information of phase distribution of all elements, radiation pattern formula of the CATG and requires users to enter coordinates of all elements in the array. The formulas of phase distribution and radiation patterns in this paper are the basis to design any CATG.

Index terms

Phased array antenna, cylindrical arrays with triangular grid (CATG), sonar application, direction-of-arrival (DOA) estimation.

1. Introduction

Cylindrical arrays of elements has been used in various applications such as navigation and sonar or as a base station antenna in a mobile communication system due to potential of 360° coverage with an omnidirectional beam or multiple beams, or a narrow beam that can be steered over 360° [1]. As a kind of cylindrical arrays, cylindrical arrays with triangular grid (CATG) have not only advantages of the cylindrical arrays but also the benefits of radiation pattern because the shape is shading to suppress the side lobes [2]. Thus, CATG has been appearing in some commercial products of sonar, for example sonar UMS 4110 and KINGKLIP of Thales group [8, 9]. However, the fact is that there is still a small amount of released academic research paying attention to this array in detail.

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Especially, determining phase distribution always plays a vital role for designers when steering the beam to any desired direction in any types of phase array antenna. With a complicated shape like the CATG, it is more difficult to find phase distribution compared with other popular types such as the linear, planar, circular arrays. In the reference investigation range of the authors, there is still no paper addressing a specific formula to synthesize phase distribution of the whole elements in the CATG.

In this paper, an effective formula of coordinate combination for each element in the CATG is proposed, which makes calculation progress of radiation pattern become much more mathematically explicit and easier compared with the direction-of-arrival (DOA) estimation using the radius-vector [4]. Moreover, with the proposed formula, detailed information of phase distribution is also identified conveniently and accurately. The validity of our proposal is confirmed by an excellent agreement in results with the method provided by the tool Sensor Array Analyzer integrated in MATLAB [7], which only generates the view of radiation pattern without letting users know the detailed phase distribution of each element and requires users to enter coordinates of all elements in the array. In addition, with considering mutual coupling effects in the paper, merits of the proposed mathematical model remain nearly unchanged for determining properties of the main beam.

2. Proposal of mathematical model

Consider the cylindrical arrays with triangular grid (CATG) as shown in Fig. 1a. The radius of the circles is \( R \), the number of elements in a circle is \( N \), and the angle between two adjacent elements is \( \Delta \theta = \frac{2\pi}{N} \). The number of the circles in the array is \( P \), and the distance between two adjacent circles is \( h \). Therefore, the total number of elements in the array is \( N \times P \). When steering the beam, the angle of active sector in each circle might be often chosen as 60°, 90° or 120° [1], and the number of active elements in each circle is \( Q (Q \leq N) \) (Fig. 1b).

The element at the point \( A_1 \) in Fig.1b is chosen as the 1st element of the array. Therefore, the \( n^{th} \) element in the \( p \)th \((1 \leq p \leq P)\) circle and the \( q^{th} \) \((1 \leq q \leq Q)\) column will satisfy the condition of \( n = Q(p - 1) + q \). The coordinates of the \( n^{th} \) element are determined as below.

\[
x_n = \begin{cases} 
R \cos \left( (q - 1) \Delta \theta \right) & \text{if } p \text{ is odd} \\
R \cos \left( (q - 1 + \frac{1}{2}) \Delta \theta \right) & \text{if } p \text{ is even} 
\end{cases} \quad (1a)
\]

\[
y_n = \begin{cases} 
R \sin \left( (q - 1) \Delta \theta \right) & \text{if } p \text{ is odd} \\
R \sin \left( (q - 1 + \frac{1}{2}) \Delta \theta \right) & \text{if } p \text{ is even} 
\end{cases} \quad (2a)
\]

\[
z_r = (p - 1)h \quad (3)
\]
Fig. 1. The cylindrical array with triangular grid (CATG) (for example, in this case, 
$N = 8, P = 4, Q = 4$) a) CATG ; b) Active elements in CATG.

The separation into two cases in formulas (1a-1b) and (2a-2b) causes difficulties in 
both controlling the phase of each element in the array and determining the radiation 
pattern since it is required to consider two separate cases of odd $p$ and even $p$. Therefore, 
a new merged solution is proposed in this paper as shown in the following formulas 
(5), (6), combining (1a) and (1b), as well as (2a) and (2b) respectively, which makes 
it much easier and more convenient in setting up a general equation for determining 
phase distribution and radiation pattern.

$$x_n = R \cos \left( q - 1 + \frac{1}{2} \left\lfloor \frac{p - 1}{2} \right\rfloor - \left\lfloor \frac{p - 1}{2} \right\rfloor \right) \Delta \theta$$

$$y_n = R \sin \left( q - 1 + \frac{1}{2} \left\lfloor \frac{p - 1}{2} \right\rfloor - \left\lfloor \frac{p - 1}{2} \right\rfloor \right) \Delta \theta$$

where $\left\lfloor t \right\rfloor$ and $\left\lceil t \right\rceil$ denote round functions toward integers of arbitrary real number $t$: 
$\left\lfloor t \right\rfloor = \min \{n \in \mathbb{Z}, n \geq t\}$ and $\left\lceil t \right\rceil = \max \{n \in \mathbb{Z}, n \leq t\}$.

With this proposed mathematical model, it is possible to consider only one general 
case to calculate radiation pattern when steering the beam to any desired direction by 
the DOA-estimation [4], and determine phase distribution of the CATG which could not 
be specified by the tool Sensor Array Analyzer of MATLAB as indicated in Section 4.

3. Determination of phase distribution and radiation pattern

Assume that the target is at point $M$, whose direction is represented by the directive 
unit vector $\vec{u} = (\cos \theta \cos \varphi, \sin \theta \cos \varphi, \sin \varphi)$. When the array is analyzed as in Fig. 1b,
it is convenient to take the phase reference \((\psi_1 = 0)\) at position \(A_1 = (R, 0, 0)\). The element \(n^{th}\) at position \(A_n = (x_n, y_n, z_n)\) is set up initial phase \((\psi_n)\) and amplitude \((a_n)\).

The path-length difference between the element \(A_n\) and \(A_1\) directing to \(M\) is [1], and it is:

\[
\Delta l = \overrightarrow{u}A_1A_n = \overrightarrow{u}(OA_n - OA_1) \tag{6}
\]

Substituting the formula (3) and the proposed formulas (4), (5) to equation (6), \(\Delta l\) becomes:

\[
\begin{align*}
\Delta l &= R \left( \cos \left( \left( q - 1 + \frac{1}{2} \left[ \frac{p-1}{2} - \left\lfloor \frac{p-1}{2} \right\rfloor \right] \right) \Delta \theta \right) - 1 \right) \cos \theta \cos \varphi \\
&+ R \sin \left( \left( q - 1 + \frac{1}{2} \left[ \frac{p-1}{2} - \left\lfloor \frac{p-1}{2} \right\rfloor \right] \right) \Delta \theta \right) \sin \theta \cos \varphi + (p - 1)h \sin \varphi \tag{7}
\end{align*}
\]

The phase difference \((\Delta \psi_n)\) between the two elements \(A_n\) and \(A_1\) is:

\[
\begin{align*}
\Delta \psi_n &= k \Delta l = kR \left( \cos \left( \left( q - 1 + \frac{1}{2} \left[ \frac{p-1}{2} - \left\lfloor \frac{p-1}{2} \right\rfloor \right] \right) \Delta \theta \right) - 1 \right) \cos \theta \cos \varphi \\
&+ kR \sin \left( \left( q - 1 + \frac{1}{2} \left[ \frac{p-1}{2} - \left\lfloor \frac{p-1}{2} \right\rfloor \right] \right) \Delta \theta \right) \sin \theta \cos \varphi + k(p - 1)h \sin \varphi \tag{8}
\end{align*}
\]

where \(k = \frac{2\pi}{\lambda}\) is wave number, and is wavelength. On the other hand, array factor (AF) of the CATG is [3]:

\[
AF(\theta, \varphi) = \sum_{n=1}^{P \times Q} a_n e^{j\psi_n} e^{j\Delta \psi_n} = \sum_{n=1}^{P \times Q} a_n e^{j(\psi_n + \Delta \psi_n)} \tag{9}
\]

To steer the main beam of the CATG to a particular point \(M_0\) at which direction is represented by the directive unit vector \(\overrightarrow{u_0} = (\cos \theta_0 \cos \varphi_0, \sin \theta_0 \cos \varphi_0, \sin \varphi_0)\), all elements in the CATG must have the same phase at \(M_0\), which means:

\[
\begin{align*}
\psi_n(\theta_0, \varphi_0) &= -\Delta \psi_n(\theta_0, \varphi_0) \\
&= -kR \cos \left( \left( q - 1 + \frac{1}{2} \left[ \frac{p-1}{2} - \left\lfloor \frac{p-1}{2} \right\rfloor \right] \right) \Delta \theta \right) - 1 \right) \cos \theta_0 \cos \varphi_0 \\
&- kR \sin \left( \left( q - 1 + \frac{1}{2} \left[ \frac{p-1}{2} - \left\lfloor \frac{p-1}{2} \right\rfloor \right] \right) \Delta \theta \right) \sin \theta_0 \cos \varphi_0 - k(p - 1)h \sin \varphi_0 \tag{10}
\end{align*}
\]

Replacing formulas (10) and (8) into (9), array factor of the CATG is obtained as below:

\[
AF(\theta, \varphi) = \sum_{n=1}^{P \times Q} a_n \left( \exp \left( jkR \cos \left( \left( q - 1 + \frac{1}{2} \left[ \frac{p-1}{2} - \left\lfloor \frac{p-1}{2} \right\rfloor \right] \right) \Delta \theta \right) - 1 \right) \times \cos \theta \cos \varphi - \cos \theta_0 \cos \varphi_0 \right) \times \exp \left( jkR \sin \left( \left( q - 1 + \frac{1}{2} \left[ \frac{p-1}{2} - \left\lfloor \frac{p-1}{2} \right\rfloor \right] \right) \Delta \theta \right) \times \sin \theta \cos \varphi - \sin \theta_0 \cos \varphi_0 \right) \times \exp (jk(p - 1)h \sin \varphi - \sin \varphi_0) \right) \tag{11}
\]
Equation (11) is convenient in order to determine phase distribution and plot radiation pattern when steering beam at specified direction. In order to take into account the mutual coupling between elements in the array, the \( P \times Q \) - element array may be seen as a \( P \times Q \) - port characterized by a \( P \times Q \) scattering matrix with the scattering coefficients whose direction is represented [3]:

\[
S_{nm} = \frac{V_m^{- \gamma}}{V_m^+} \mid_{V_l^+ = 0, l \neq m} \quad \text{for} \ n, m, l = 1, 2, \ldots, P \times Q. 
\]

where \( V_m^+ \) and \( V_m^- \) are the wave amplitude of, respectively, the incident and reflected waves at the element \( n \). Thus, when taking the mutual coupling effects into account and representing the incident wave by \( V_n^+ = a_n \exp(j\psi_n(\theta_0, \varphi_0)) \), equation (11) becomes:

\[
\begin{align*}
\Gamma_n(\theta_0, \varphi_0) &= \frac{V_n^-}{V_n^+} = \frac{\sum_{m=1}^{P \times Q} S_{nm}V_m^+}{V_n^+} \\
&= \sum_{m=1}^{P \times Q} S_{nm} \exp\left(-jkR\left(\cos\left((q_m - 1 + \frac{1}{2} \left\lfloor \frac{p_m-1}{2} \right\rfloor - \left\lfloor \frac{p_m-1}{2} \right\rfloor \right)\Delta\theta - 1\right) \cos \theta_0 \cos \varphi_0 \right.
\right.
\end{align*}
\]

\[
\times \sin \theta_0 \cos \varphi_0 - jk(p_m - 1)h \sin \varphi_0 \\
\times \exp\left(\left(\cos\left((q - 1 + \frac{1}{2} \left\lfloor \frac{p-1}{2} \right\rfloor - \left\lfloor \frac{p-1}{2} \right\rfloor \right)\Delta\theta - 1\right) \cos \theta_0 \cos \varphi_0 \right.
\right.
\]

\[
\times \exp\left(\left(\cos\left((q - 1 + \frac{1}{2} \left\lfloor 2 - \frac{2}{2} \right\rfloor - \left\lfloor 2 - \frac{2}{2} \right\rfloor \right)\Delta\theta - 1\right) \cos \theta_0 \cos \varphi_0 + k(p - 1)h \sin \varphi_0 \right)
\]  

Equations (13) and (14) give mathematical model for designers to determine radiation pattern when mutual coupling between elements in the CATG is considered.

4. Simulation results

We consider an example of the CATG applied in a sonar system, which has 24 elements (\( N = 24 \)) on a circle and 16 circles (\( P = 16 \)) in total. If the active sector is chosen as one third of the circle, the number of active elements on a circle is 8 (\( Q = 8 \)), and the total number of active elements in the array is \( 8 \times 16 = 128 \) elements. The angle
between two adjacent elements is $\Delta \theta = 15^\circ$ and the angle blocked by the 8 elements is determined $\theta_{\text{active}} = 105^\circ$. The simulation is implemented at frequency $f = 30$ kHz, while the sound speed in sea water is 1500 m/s, then the distance between two adjacent circles is chosen $h = \frac{1}{1 + \sin 45^\circ} \lambda \approx 29.29\, (mm)$ (steering beam up to $45^\circ$ [3]). The radius of a circle is chosen as $R = 5\lambda = 250\, (mm)$ [1], and the desired steering angles in
azimuth and elevation planes are $\theta_0 = 60^\circ$ and $\varphi_0 = 0^\circ$ respectively. We assume that amplitude distribution in the array is equal, each element has the isotropic radiation pattern, and there is no mutual coupling between them in the array.

The results of the proposed method and the tool Sensor Array Analyzer of MATLAB are shown in Fig. 2. The cut angle in elevation plane is $\varphi_0 = 0^\circ$ (Fig. 2a), and the cut angle in azimuth plane is $\theta_0 = 60^\circ$ (Fig. 2b). In each figure, there are a dashed curve resulting from the proposed method and a solid curve resulting from the tool Sensor Array Analyzer of MATLAB for comparison. It is easy to confirm in these figures an excellent agreement between the dashed and solid curves, which means that the array factor radiation pattern generated by formula (11) from our proposal is definitely similar to that raised automatically by the tool Sensor Array Analyzer of MATLAB. Therefore, the reliability of our proposal is validated totally, and moreover, in our method, the desired phase of each element in the array is simply derived by using formula (10) as demonstrated in Fig. 3, while the tool Sensor Array Analyzer of MATLAB requires users to enter coordinates of all elements in the array and does not provide the users with any information of phase distribution.

In order to obtain simulation results when accounting mutual coupling, equations (13) and (14) are used. Determination of the scattering coefficients in equation (13) may be implemented by measurements [1, 3] or mathematical formulas. Consider the CATG with transducer elements used as references [8, 9], which consist of short cylindrical transducers distributed in surface of the array. We assume that every element is match-connected to a voltage source ($S_{nn} = 0$) [3]. In order to calculate reflected wave amplitude, scattering cross-section known in radar as the radar cross-section (RCS) is used [5]. When the diameter of each transducer is chosen as $D_0 = \frac{1}{3} \approx 16.67(mm)$, length of this cylinder is chosen as $H_0 = \frac{D_0}{10} \approx 1.7(mm)$, the maximum value of RCS of each element is $\sigma = \frac{\pi D_0 H_0^2}{\lambda}$ [6]. With the radar cross-section model, the maximum absolute values of the scattering coefficients are estimated by $|S_{nm}|_{\text{max}} = \sqrt{\frac{\sigma}{4\pi r^2_{nm}}}$. 

Fig. 3. Phase distribution generated by formula (10)
Fig. 4. Array factor when taking mutual coupling into account a) In azimuth plane ($\varphi = 0^\circ$); b) In elevation plane ($\theta = 60^\circ$)

Where $r_{nm}$ is distance between the element $n_{th}$ and the element $m_{th}$ of the array. Because the phase values of the scattering coefficients are proportional the distances, the complex value of the scattering coefficients are estimated by

$$ S_{nm} = |S_{nm}| \exp(-jkr_{nm}) $$

(15)
With using estimations of the scattering coefficients determined by the maximum absolute values, the scan reflection coefficients and radiation pattern are calculated by equations (13) and (14).

The results of two cases of with and without mutual coupling are shown in Fig. 4. The cut angle in elevation plane is $\varphi = 0^\circ$ (Fig. 4a), and the cut angle in azimuth plane is $\theta = 60^\circ$ (Fig. 4b). In each figure, there are a solid curve for the case without mutual coupling and a dashed curve for the case with mutual coupling.

It could be seen that in this example, mutual couplings between the elements have the major effects on the side lobes, but the direction and the beamwidth of the main beam remain relatively stable. The simulation results for the CATG are the same with the released results for some cylindrical arrays with rectangular grid in the reference [1]. Therefore, with a given phase distribution, properties of the main beam will not be affected too much, that is to say, formula (10) can also be used in the case of taking mutual couplings into account in this specific case, and the availability of formulas (4)-(5) is still maintained. With other cases in which mutual couplings yield in other complex forms, it is required to consider carefully the mathematical formation of mutual couplings in each case.

5. Conclusion

In this study, with a simple proposal of coordinate combination, determination of radiation pattern and phase distribution in cylindrical arrays with triangular grid (CATG) becomes much more mathematically explicit. The validity and merits of our proposal are verified by comparing the obtained results with those raised by the tool Sensor Array Analyzer of MATLAB. The simulation results also indicate that in a specific case of considering mutual couplings between elements in the array, the proposed formulas is still available for deriving the properties of the main beam.

References


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CÔNG THỨC TỌA ĐỘ TỔNG QUÁT CHO THIẾT KẾ CÁC ANTEN MẠNG PHA HÌNH TRỤ VỚI CÁC PHẦN TỬ ĐẠN CHÉO

Tóm tắt

Bài báo đề xuất một mô hình toán học hợp nhất tọa độ trong anten mạng pha hình trụ với các phần tử đan chéo, một trong những dạng anten được sử dụng trong các ứng dụng sonar. Với giải pháp đề xuất, phân bố pha trong mạng anten này có thể được xác định một cách nhanh chóng để tính toán giản đồ hướng khi muốn điều khiển cánh sóng đi đến hướng bất kỳ. Các kết quả được khảo sát trong cả hai trường hợp có và không có ảnh hưởng tương hỗ giữa các phần tử trong mạng. Đồ thị của phần dự đoán của công cụ Sensor Array Analyzer của Matlab. Tuy nhiên, công cụ của Matlab không cung cấp thông tin về phân bố pha của các phần tử, công thức xác định phân bố pha và giản đồ hướng được đưa ra trong bài báo là cơ sở để thiết kế anten mạng pha hình trụ với các phần tử đan chéo.